Pitch, Pitch Interval and Pitch Ratio Representation

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Abstract

This document describes how pitch can be represented using various units. More specifically it documents how a software program to analyse pitch in music, Tarsos, represents pitch. Tarsos[3] can be downloaded on http://tarsos.0110.be. This document contains definitions of and remarks on different pitch and pitch interval representations. For good measure we need a definition of pitch, here the definition from [1] is used: *The pitch frequency is the frequency of a pure sine wave which has the same* perceived *sound as the sound of interest.* For remarks and examples of cases where the pitch frequency does not coincide with the fundamental frequency of the signal, also see [1]. In this text pitch, pitch interval and pitch ratio are briefly discussed.

1 Pitch & Pitch Interval Representation

Since we are interested in a frequency or frequency interval Hertz (Hz), oscillations per second, seems the most appropriate unit. When working with sound this is not always the case. For humans the perceptual distance between 220Hz and 440Hz is the same as between 440Hz and 880Hz. A pitch representation that takes this logarithmic relation into account is more practical for some purposes. Luckily there are a few:

MIDI Note Number
: The MIDI standard defines note numbers from 0 to 127, inclusive. Normally only integers are used but any frequency f in Hz can be represented

with a fractional note number n using equation 1.

$$n = 69 + 12\log_2(\frac{f}{440}) \quad (1)$$
$$n = 12 \times \log_2(\frac{f}{r}) \ ; \ r = \frac{440}{2^{(69/12)}} = 8.176\text{Hz} \quad (2)$$

Rewriting equation 1 to 2 shows that MIDI note number 0 corresponds with a reference frequency of 8.176Hz which is C_{-1} on a keyboard with A_4 tuned to 440Hz. It also shows that the MIDI standard divides the octave in 12 equal parts.

To convert a MIDI note number n to a frequency f in Hz one of the following equations can be used.

$$f = 440 \times 2^{(n-69)/12} \quad (3)$$
$$f = r \times 2^{(n/12)} \text{ with } r = 8.176\text{Hz} \quad (4)$$

Using pitch represented as fractional MIDI note numbers makes sense when working with MIDI instruments and MIDI data. Although the MIDI note numbering scheme seems oriented towards western pitch organization (12 semitones) it is conceptually equal to the cent unit which is more widely used in ethnomusicology.

Cent

Ellis introduced the nowadays widely accepted cent unit. To convert a frequency f in Hz to a cent value c relative to a reference frequency r also in Hz.

$$c = 1200 \times \log_2(\frac{f}{r}) \quad (5)$$

With the same reference frequency r equations 5 and 2 differ only by a constant factor of exactly 100. In an environment with pitch representations in MIDI note numbers and cent values it is practical to use the standardized reference frequency of 8.176Hz.

To convert a frequency f in Hz to a cent value c relative to a reference frequency r also in Hz.

$$f = r \times 2^{(c/1200)} \quad (6)$$

Savart & Millioctaves

Divide the octave in 301.5 and 1000 parts respectively, which is the only difference with cents.

2 Pitch Ratio Representation

Pitch ratios are essentially pitch intervals, an interval of one octave, 1200 cents equal to a frequency ratio of 2/1. To convert a ratio t to a value in cent c:

$$c = \frac{1200 \ln(t)}{\ln(2)} \tag{7}$$

The natural logarithm, the logarithm base e with e being Euler's number, is noted as ln. To convert a value in cent c to a ratio t:

$$t = e^{\frac{c \ln(2)}{1200}} \tag{8}$$

Further discussion on cents as pitch ratios can be be found in appendix B of [2]. There it is noted that:

> There are two reasons to prefer cents to ratios: Where cents are added, ratios are multiplied; and it is always obvious which of two intervals is larger when both are expressed in cents. For instance, an interval of a just fifth, followed by a just third is $(3/2)(5/4) = 15/8$, a just seventh. In cents, this is $702 + 386 = 1088$. Is this larger or smaller than the Pythagorean seventh 243/128? Knowing that the latter is 1110 cents makes the comparison obvious.

3 Conclusion

The cent unit is mostly used for pitch interval representation while the MIDI key and Hz units are used mainly to represent absolute pitch. The main difference between cent and fractional MIDI note numbers is the standardized reference frequency. In our software platform Tarsos we use the exact same standardized reference frequency of 8.176Hz which enables us to use cents to represent absolute pitch and it makes conversion to MIDI note numbers trivial. Tarsos also uses cents to represent pitch intervals and ratios.

4 Bibliography

[1] Philip McLeod. *Fast, accurate pitch detection tools for music analysis.* PhD thesis, University of Otago. Department of Computer Science, 2009.

[2] William Sethares. *Tuning Timbre Spectrum Scale.* Springer, 2 edition, 2005.

[3] Joren Six and Olmo Cornelis. Tarsos - a Platform to Explore Pitch Scales in Non-Western and Western Music. In *Proceedings of the 12th ISMIR Conference*, 2011.